Global three-dimensional optimal perturbations in a Blasius boundary layer

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Abstract The three-dimensional global optimal and near-optimal perturbations in a flat-plate boundary layer are studied by means of an adjoint-based optimization, and their non-linear evolution is investigated by means of DNS.

1 Introduction

Since the early observations [1], many studies have been dedicated to the interior structure of turbulent spots, their shapes, spreading rates, and mechanisms of growth [2]. However, minor efforts have been dedicated so far to identify the initial, localised states which most effectively bring the flow on the verge of turbulent transition via the formation of a spot. In this work a new attempt is made to identify optimal initial disturbances capable to induce breakdown to turbulence in a boundary layer. The optimization is not restricted to an initial state (at $x = 0$ or $t = 0$) characterised by a single wavenumber, but it considers a wave packet, localised in the streamwise (and eventually spanwise) direction, formed by the superposition of monochromatic waves. In order to assess whether such an optimal localised flow state is effective in provoking breakdown, direct numerical simulations (DNS) are then performed, highlighting the importance of non-linear effects which lie at the heart of the initiation of a turbulent spot.

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2 Problem formulation

The linear behaviour of a perturbation, $\mathbf{q} = (u, v, w, p)^T$, evolving in a laminar incompressible flow past a flat-plate is studied by employing the 3D Navier-Stokes (NS) equations linearized around the 2D steady state $\mathbf{Q} = (U, V, 0, P)^T$. Dimensionless variables are defined with respect to the inflow boundary layer displacement thickness δ^* and to the freestream velocity U_∞, so that the Reynolds number is $Re = U_\infty \delta^*/\nu$. The linearized NS equations are integrated by a fractional step method using a second-order-accurate centered space discretization on a staggered grid [3]. In order to investigate the global optimal perturbations, a Lagrange multiplier technique is employed, where the perturbation kinetic energy integrated over the whole domain is chosen as the objective function. By imposing the linearized NS equations as a constraint and following a procedure similar to the one described in [4], the adjoint equations are recovered and integrated in time together with the direct equations until convergence to the optimal solution is achieved. Finally, the non-linear evolution of the optimal perturbation is investigated by means of DNS.

3 Results and discussion

Computations have been performed at $Re = 610$, for a domain of dimensions $L_x = 400$, $L_y = 20$, $L_z = 10.5$, where the value of L_z has been chosen in order to obtain the largest amplification. The energy gain, $G(t) = E(t)/E(0)$, has been found to reach approximately $G(t) \approx 736$ at time $t_{max} \approx 247$. The optimal spatially localised initial disturbance is characterized by a counter-rotating vortex pair in the $y - z$ plane, reminiscent of the one predicted by the local optimization for a perturbation with the streamwise wavenumber $\alpha = 0$. Indeed, in the present case, a modulation is found in the x direction, the perturbation being composed by upstream-elongated structures with x-alternated-sign velocity components, as shown in Figure 1 (a). The time evolution of such an optimal solution shows that the perturbation is tilted in the mean flow direction by means of the Orr mechanism [5], while being amplified by the lift-up mechanism, resulting at the optimal time in streaky structures alternated in the x direction (see Figure 1 (b)).
Similar structures have been found for lower Reynolds numbers and different domain lengths. In particular, for large spanwise domain lengths, the optimal perturbation results to be a single-spanwise-wavenumber perturbation similar to the one in Figure 1 extended in the whole spanwise direction. However, in nature, disturbances are always characterized by a wide spectrum of frequencies, and often are localized in wave packets, so that a localized perturbation would be more suitable to represent the disturbance which is more likely to lead the flow to a chaotic behaviour. Therefore, an artificial wave packet has been constructed for $L_z = 180$ by multiplying the optimal single-wavenumber perturbation by an envelope which varies like $\exp(-z^2/L_z)$, chosen as the initial configuration for the optimization process. A partially optimized perturbation, shown in Figure 3 (a), has been extracted at the level

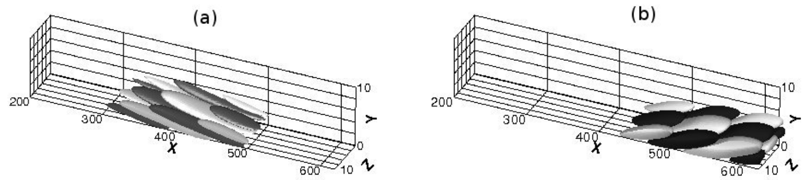

Fig. 1 Iso-surfaces of the streamwise optimal perturbation for $Re = 610$ at $t = 0$ and $t = t_{max}$.

of convergence $e_1 = (E(t)^{(n)} - E(t)^{(n-1)})/E(t)^{(n)} = 10^{-3}$. Analyzing such an intermediate solution, it is possible to notice that the perturbation is still localized in the spanwise direction, although its shape is changed. In particular, the streak-like structures at the edge of the wave packet is inclined with respect to the z axis, resulting in oblique-like waves bordering the wave packet. Furthermore, such a *near-optimal* solution, although different in the spanwise direction with respect to the optimal one, can be amplified up to very high values of the energy gain, reaching a value which differs of less than 1% from the optimal one ($(G(t)_{e_1} = 728$).

In order to investigate the capability of the three-dimensional optimal perturbation to induce transition in a boundary-layer flow, non-linear simulations are performed and compared to those arising from the evolution of the local optimal (with $\alpha = 0$) and suboptimal (with $\alpha = 2\pi/L_z$) disturbances. Figure 2 shows the mean skin friction coefficient measured in the simulations initialized with three initial energies, $E_{0(a)} = 0.5$, $E_{0(b)} = 2$ and $E_{0(c)} = 10$, as well as the theoretical trend of the laminar and turbulent skin friction coefficient in a boundary layer. As shown by the curves, the global optimal disturbance is the most effective in inducing transition, followed by the local suboptimal one. Such results confirm those in [6], in which the suboptimal perturbation is found more effective in inducing transition than the local optimal one, but also assesses that a localized x-modulated array of elongated structures is able to lead a subcritical boundary layer to chaos for an even lower value of the initial energy.

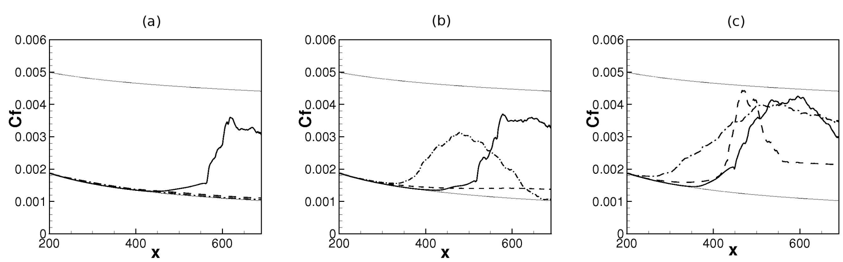

Fig. 2 Mean skin friction coefficient: theoretical values for a laminar (lowest dotted line) and turbulent (highest dotted line) boundary layer and computed values for the considered flow perturbed with the global three-dimensional optimal (solid line), the local optimal (dashed line), and the suboptimal (dashed-pointed line), at $E_{0_a} = 0.5$ (a), $E_{0_b} = 2$ (b) and $E_{0_c} = 10$ (c).

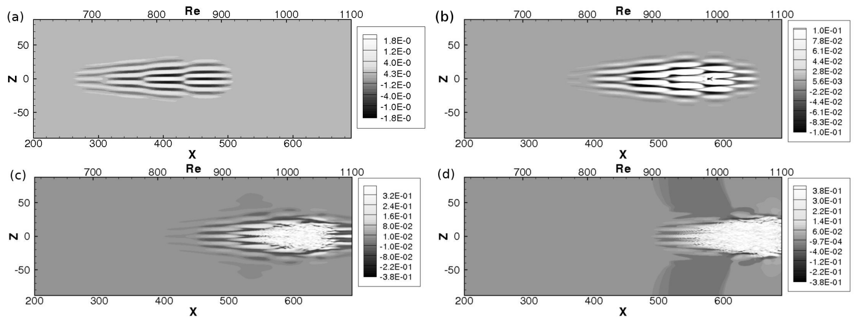

Fig. 3 Contours of the evolution of the streamwise near-optimal velocity for $y = 1$ at $T = 0$ (a), $T = 220$ (b), $T = 330$ (c) and $T = 420$ (d).

In order to generalize such a result to larger and more realistic domain lengths, the non-linear evolution of the *near-optimal* wave packet has been investigated. The perturbation has been extracted at three instants of time from a DNS computation with $E_0 = 0.5$. Figure 3 (b) shows that at $t = 220$ the wave packet has been convected downstream, amplifying itself and saturating its energy, so that the streaky structures seem to partially merge, and two kinks appear at the leading edge of the most amplified streaks. It has been observed that, at this time both the scenarios of quasi-sinuous and quasi-varicose breakdown (see [7]) could be identified in the present case due to the staggered arrangement of the streaks. As a result, at least four streaks experience breakdown at the same time, explaining the efficiency of the global optimal and near-optimal perturbation in inducing transition. Due to streak breakdown, at $t = 330$ the most amplified elongated structures in the middle of the wave packet have already experienced transition, (see Figure 3 (c)), and at a sufficiently large time the linear wave packet has totally disappeared and the disturbance has taken the form of a localized *turbulent spot* (see Figure 3 (d)).
The optimal and near-optimal wave packets computed by means of the three-dimensional direct-adjoint optimization, which have been found more effective than the local optimal perturbations in inducing transition, could thus represent a linear precursor of a *turbulent spot*, and the transition mechanism here investigated could represent an optimal way to transition.

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