SENSITIVITY AND FORCING RESPONSE IN SEPARATED BOUNDARY-LAYER FLOW

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Objectives

1. To analyse the space-time dynamics in non parallel separated flows with a global linear stability approach,
2. To study the sensitivity and the optimal response forcing.

1. flat plate boundary-layer flow,
2. incompressible regime at low Reynolds number,
3. Separated zone is generated by normal velocity suction profile.

Movie from IAG/Stuttgart University, with courtesy of U. Rist.
1. Theoretical background,
2. Base flow: configuration & computation,
3. Results:
   1. Asymptotic dynamics,
   2. Transient dynamics,
   3. Sensitivity by pseudo-spectrum approach,
   4. Optimal forcing response to harmonic real forcing.
4. Conclusions
General aspects:

- Instantaneous flow: \( Q(x, y, z, t) = \overline{Q}(x, y) + \varepsilon q(x, y, z, t) \) with \( \varepsilon << 1 \)
- base flow: \( \overline{Q}(x, y) = (U, 0, P)^t \), steady and two-dimensional.
- small perturbation: \( q(x, y, z, t) = (u, p)^t (x, y, t) e^{i\beta z} \).

Linearized perturbated evolution operator:

\[
\begin{align*}
B \frac{\partial q}{\partial t} &= Aq + fe^{i\sigma t}, \\
q_0 \text{ the initial condition}
\end{align*}
\]

- the operator \( A \) and \( B \) are deducted from the linearized Navier-Stokes equations,
- \( f \) is a spatially localized forcing term,
- \( \sigma \) is the forcing frequency.
Theoretical background: global stability approach (2)

Global modes decomposition:

- The solution of the evolution equation is decomposed into a global modes basis where the normal and streamwise directions are taken as eigendirections:

\[
q(x, y, z, t) = \sum_{n=1}^{N} K_n \hat{q}_n(x, y) e^{i(\beta z - \Omega_n t)} \text{ and }
\]

\[
q_0(x, y, t = 0) = \sum_{n=1}^{N} K_n \hat{q}_n(x, y)
\]

where \(\Omega_n\) and \(\hat{q}_n\) solutions of large eigenvalues problem from LNS eq:

\[
(A - \Omega B) \hat{q} = 0
\]
Base Flow

Reference scales and configuration.

- \( \delta^* \) at \( x = 0 \) and maximum exterior velocity \( U_e \).
- \( Re_{\delta^*} = 200 \). \( L_x = 500 \) and \( L_y = 25 \).

Boundary conditions. Suction profiles.

- Suction profiles. Three base flows D1, D2 and D3.
Results: asymptotic global stability analysis, $\beta = 0$

Global spectrum ($N_x = 250$, $N_y = 48$). Krylov subspace: 850

- $L_x = 400$. Three kinds of global modes F1, F2 and F3.
- Globally stable.

Similar studies:

U. Ehrenstein and F. Gallaire.
On two dimensional temporal modes in spatially evolving open flows: the flat-plate boundary layer.
Theoretical background: transient growth analysis

- Transient growth related to the non normality of the global operator. [1, 2].
- To maximize the quantity $G(t)$:

$$< u_i, u_j >_E = \int_0^{L_y} \int_0^{L_x} (u_i^* u_j + v_i^* v_j) \, dx \, dy$$

$$G(t) = \max_{u_0} \frac{\| u(t) \|_2^2}{\| u_0 \|_2^2} = s v_1 \left( \| F \exp(tD) F^{-1} \|_2 \right)^2$$

with $D, D_{l,k} = -\delta_{lk} i \Omega_k$ and $M = F^* F$, $M_{i,j} = < u_i, u_j >_E$

Precursor studies:

- C. Cossu and J.-M. Chomaz.
  Global measures of local convective instabilities.

- P.J. Schmid
  Nonmodal stability theory.
Results: Transient growth analysis, $\beta = 0$

Transient growth for the bubble D3.

Convergence of the global modes basis.

First phase: High increasing of the energy associated to the optimal perturbation. Orr and Kelvin-Helmholtz mechanisms induced by the shear layer. Non normality of normal and streamwise direction.

Second phase: Energy decreasing - asymptotic dynamics reached.

Similar studies:

˚Akervik et al.

Ehrenstein, U. and Gallaire, F.
Partial conclusions & new motivations

**Partial conclusion**

- This analysis have shown, in fully non-parallel context (global modes analysis) that the linearized dynamics inside a laminar separation bubble is strongly non-normal (high transient growth - Orr & K-H mechanisms) and mainly dominated by convective instability for the long times.

**New motivations**

- It seems interesting to develop a strategy based on the non-normality by means of global modes in order to describe the convective instability related to the response of an harmonic forcing of the laminar separation bubble.

- Two way are investigated:
  1. Study of basin of sensitivity of each global modes for fixed forcing: pseudo-spectrum computation.
  2. Study of optimal response forcing at harmonic forcing: optimal resolvant $R(\Omega_f)$ computation.
Theoretical background: sensitivity analysis.

- In this "global" framework the existence of convective waves as a response to an harmonic forcing is related to a near resonant which can be explained by the non normality of the operator associated to the initial value problem.

- This particularity can be introduced by the analysis of the pseudo-spectrum of the operator:

\[ B \frac{\partial q}{\partial t} = Aq + f e^{i\sigma t} \]

- General and asymptotic solutions:

\[ \tilde{q} = q_0 e^{Dt} + \hat{f} e^{i\sigma t} \text{ when } \tilde{q} \xrightarrow{t \to \infty} \hat{f} e^{i\sigma t} \text{ with } (i\sigma B - A)\hat{f} = f \]

- Pseudo-spectrum evaluates the sensitivity to real harmonic forcing

\[ \lambda_\epsilon = \{ \sigma \in \mathbb{C}, \|P(\sigma)\| \geq \epsilon^{-1} \} \text{ with } P(\sigma) = (i\sigma B - A)^{-1}, \]

- The system being too large to solve the entire pseudospectrum \( \lambda_\epsilon \), the Hessenberg matrix resulting from the Arnoldi calculation is used as an approximation.
Results: sensitivity analysis, $\beta = 0$

- Strong sensitivity for convective modes,
- modes most sensitive are closed to $\Omega_r \approx 0.08$, 
  $\Rightarrow$ strong sensitivity can triggered a vortex shedding phenomenon.

Instantaneous vorticity: $f \approx 0.08/2\pi$

- Good comparison with 2D-DNS results (forcing by a white noise):
  dominant $\Omega_{DNS} \simeq 0.0851$ and $\Omega_{stab} = 0.0849$. 
We will now investigate more precisely the shape of the response and the mechanism associated to the real harmonic forcing.

The response for long times to an harmonic forcing associated to the perturbation can be formulated as a summation of temporal modes:

$$\mathbf{B}\frac{\partial \mathbf{q}}{\partial t} = \mathbf{Aq} + \mathcal{F},$$

with harmonic forcing: $$\mathcal{F}(\mathbf{x}, t) = \sum_k f_k \hat{q}_k(\mathbf{x}) e^{-i\Omega_f t}$$

Flow response is given by:

$$\mathbf{q}(\mathbf{x}, t) = \sum_k \left( K_{k,h} e^{-i\Omega_k t} - \frac{if_k}{(\Omega_f - \Omega_k)} e^{-i\Omega_f t} \right) \hat{q}_k(\mathbf{x})$$

The maximum response of the separated flow to a forcing is given by:

$$\mathcal{R}(\Omega_f) = \max_{\mathcal{F}} \frac{\|\mathbf{q}\|_E}{\|\mathcal{F}\|_E} = \|\mathbf{F}\mathbf{D}_f\mathbf{F}^{-1}\|_2 = \text{sv}_1 (\mathbf{F}\mathbf{D}_f\mathbf{F}^{-1})$$
Results: optimal forcing response (1)

Evolution of the value of $R(\Omega_f)$ with the number of temporal modes for $L_x = 400$

- Base convergence around $N \approx 500$,
- No domain $(L_x, L_y)$ dependency,
- Optimal response frequency: $\Omega_f \approx 0.07$,
- Weak sensitivity at high frequency,
Results: optimal forcing response (2)

optimal initial forcing for $\Omega_f = 0.07$

Comparison at $y = 2$ and $\Omega_f = 0.07$ between the linearized DNS and the 500 temporal modes summation

Streamwise velocity:

optimal forcing for $t = 400$ and $\Omega_f = 0.07$

- For very short time, Orr mechanism is present (the spatial evolution is similar to that observed in the transient growth study).
- Linearized DNS, initialized with optimal initial forcing, gives similar results:
  - same velocity, same spatial evolution.
- For a long time, the spatial structure for the perturbation is close to Tollmien-Schlichting waves.
Conclusions and prospects

Conclusions

- Possibility to analyse the transient and asymptotic behaviour of the separated flow with a reduced model based on global modes.
- At this Reynolds number, $Re_{δ^*} = 200$, two different mechanisms: high 2D transient mechanism ($G_{max} \approx 10^8$, $t_{max} \approx 350$) and a 3D centrifugal mechanism, what is the dominant mechanism?
- Sensitivity of the complete spectrum: high sensitivity for the convective modes. This high sensitivity can triggered a vortex shedding phenomenon based on the Kelvin-Helmholtz instability.
- Optimal response to the forcing: the resolvant presents a peak in the vicinity of the most sensitive modes ($Ω \approx 0.07$).

Prospects

- Taking into account the non linear aspects,
- Control strategy ? (for the short time ? for the long time ?),
- Three dimensional transient growth, lift-up-effect ?
- Direct-adjoint strategy, comparison with the reduced model ?
- Direct numerical simulation and experiments.
Numerical methods

Base flows:

- Incompressible 2D Navier Stokes equations in $(\omega, \psi)$:
  \[
  \frac{\partial \omega}{\partial t} + u \frac{\partial \omega}{\partial x} + v \frac{\partial \omega}{\partial y} = \frac{1}{Re} \left( \frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2} \right)
  \]
  and $\Delta \psi = \omega$,

- Second order finite differences scheme.
- A semi-implicit AB2/CN for the time integration.
- Direct Solver for implicit system and Poisson equation.

Eigenvalues problem:

- Chebyshev/ Chebyshev collocation spectral.
- Shift and Invert Arnoldi algorithm. Large Krylov subspace.
Results: asymptotic global stability analysis, $\beta \neq 0$

Spectrum. $\beta = 0.08$

Strongly dependant of the bubble shape.

Isovorticity contours: $M_{GS}$ and $M_{Gort}$

$\Longrightarrow$ Centrifugal origin.